General notes

Several tasks contain multiple choice or true/false questions. While it may be possible that you know how to perform certain operations mentally, I need you to mark down the essential steps you took to be able to reproduce what you did. This is mainly for two reasons:

- If your end result happens to be wrong, I could not give you partial scores;
- there is no way for me to tell whether you possibly obtained your result by chance or fraud.

1 Propositional logic

1.1 Tautology et altres (18 pts)

Given the following formulas

$$\begin{aligned} f_1 &:= p \wedge s \wedge q \vee (p \rightarrow \neg s \vee \neg q), \\ f_2 &:= (s \vee p \wedge \neg t \wedge t \leftrightarrow q) \wedge \neg(q \rightarrow s), \\ f_3 &:= (s \vee s \wedge p \wedge r \rightarrow \neg q \wedge t) \vee s \wedge q \vee s \wedge \neg t, \end{aligned}$$

check all correct table cells:

	f_1	f_2	f_3
tautology			
satisfiability			
contradiction			
contingency			

1.2 Conjunctive normal form (10 pts)

Transform

$$\neg(p \wedge q \vee s) \vee r \wedge t$$

into CNF.

1.3 Proof (18 pts)

Four scientists are nominated for an achievement award. The award committee announces that

a) Wittgenstein receives an award if and only if Turing receives one or Goedel does not;

b) Einstein receives an award if and only if also Wittgenstein receives one;

c) Einstein, Turing, and Wittgenstein receive an award.

After the committee's announcement, Goedel claims that

d) if he receives an award, then Turing receives one only if Einstein does, too.

Which of the following is true considering the announcements a) to c):

1) Goedel is right.
2) Goedel is wrong.
3) Goedel's claim can neither be proven nor disproven as the committee's announcements are inconsistent.
4) Goedel's claim can neither be proven nor disproven as the committee's announcements are not informative enough.

2 First-order logic

2.1 Clausal normal form (12 pts)

Given the signature $\Sigma_1 = \langle V, F, P, \text{arity}\rangle$ with

$$V = \{u, v, w, x, y, z\}$$

$$F = \{f, g, h\}$$

$$P = \{p, q, r\}$$

$$\text{arity} = \{\langle f, 3\rangle\ \langle g, 0\rangle\ \langle h, 1\rangle\ \langle p, 2\rangle\ \langle q, 0\rangle\ \langle r, 2\rangle\}$$

turn the following formulas into clausal normal form:

$$f_1 := \forall x \exists y (q \vee q \rightarrow q \wedge p(h(y), h(x)))$$

$$f_2 := q \rightarrow \forall x \exists y (\neg \exists z (p(y, h(g)) \wedge r(z, g)))$$

$$f_3 := \neg f_2$$

2.2 Unification (12 pts)

Using the above introduced signature Σ_1, determine, if possible, a unifier for the following SSEs:

$$E_1 := \{v \doteq g, g \doteq f(h(w), y, x), y \doteq w, x \doteq w, u \doteq h(v)\}$$

$$E_2 := \{h(h(g)) \doteq x, y \doteq z, f(z, u, w) \doteq f(z, g, g), x \doteq h(v), f(w, w, u) \doteq f(u, z, z)\}$$

$$E_3 := \{x \doteq v, x \doteq z, z \doteq z, y \doteq h(z), u \doteq u, w \doteq f(h(z), f(y, f(y, u, f(z, u, u)), f(h(x), v, h(v))), v)\}$$

2.3 Proof (10 pts)

Prove that

$$\forall x \exists y (p(x) \wedge q(y)) \rightarrow \forall x, y (p(x) \vee q(y))$$

is universally valid.

3 Prolog (20 pts)

The natural logarithm of a real number $x > 1$ can be expressed as

$$\log(x) = \int_1^x \frac{1}{y} \mathrm{d}y.$$

A straightforward way to compute this integral in a computer program would be to use the equivalence

$$\int_1^x \frac{1}{y} \mathrm{d}y = \lim_{\varepsilon \to 0} \sum_{y \in X(x,\varepsilon)} \frac{\varepsilon}{y} \quad \text{with} \quad X(x,\varepsilon) = \{x - i\varepsilon | i \in \mathbb{I} \wedge x - i\varepsilon \geq 1\}.$$

A student comes up with the following Prolog program to implement the above sum formula using a static value $\varepsilon = 0.0001$:

```
log(X,0):-X>=1,X<1+0.0001.
log(X,Y):-log(A,B),A is X-0.0001,Y is B+0.0001/X.
```

a) Explain what happens when you try to run the program with the query

```
log(10,Y).
```

b) Rewrite the program so that it produces the desired outcome for the query in a) including the correct behavior when hitting the `;` key.

c) Explain why the first rule of the program cannot simply read

```
log(1,0).
```

d) Alter the program from b) to use a generic parameter ε in all program rules. This parameter is to be defined once in an additional rule at the top of the program.

e) Determine the output of the program from d) when executing the following query (use the parameter $\varepsilon = 1$)

```
log(3,Y).
```