General notes

Several tasks contain multiple choice or true/false questions. While it may be possible that you know how to perform certain operations mentally, I need you to mark down the essential steps you took to be able to reproduce what you did. This is mainly for two reasons:

- If your end result happens to be wrong, I could not give you partial scores;
- there is no way for me to tell whether you possibly obtained your result by chance or fraud.

1 Propositional logic

1.1 Tautology et altres (9 pts)

Given the following formulas

$$\begin{aligned} f_1 &:= \neg D \vee A \to D \wedge \neg D \wedge B, \\ f_2 &:= C \vee A \wedge B \vee D \vee \neg D \wedge \neg C, \\ f_3 &:= \neg A \vee E \wedge D \vee B \to C, \end{aligned}$$

check all correct table cells:

	f_1	f_2	f_3
tautology			
satisfiability			
contradiction			
contingency			

1.2 Conjunctive normal form (5 pts)

Transform

$$E \vee B \to A \to C \to D$$

into CNF.

1.3 Proof (9 pts)

Carl Friedrich Gauss is studying orbits of planets. He finds that for a certain summer night,

a) Venus is visible or both Venus and Jupiter;

b) if Saturn is visible if Jupiter is, then Jupiter is not;

c) Venus is visible if Venus and Jupiter are.

Gauss concludes

d) Jupiter and Saturn are visible if Venus is.

Which of the following is true considering the axioms a) to c):

1) d) can be proven;

2) d) can be disproven;

3) d) can neither be proven nor disproven.

2 First-order logic

2.1 Evaluation of formulas (6 pts)

We are given the signature $\Sigma = \langle V, F, P, \text{arity}\rangle$ with

$V = \{x, y, z\}$

$F = \{\mathtt{0}, \mathtt{1}, \times, -\}$

$P = \{=, >\}$

$\text{arity} = \{\langle \mathtt{0}, 0\rangle \, \langle \mathtt{1}, 0\rangle \, \langle \times, 2\rangle \, \langle -, 2\rangle \, \langle =, 2\rangle \, \langle >, 2\rangle\}$

and the structure

$$S = \langle U, J\rangle$$

as follows:

1. $U = \{a, b\}$
2. $0^J = a$
3. $1^J = b$
4. $\times^J = \{\langle\langle a, a\rangle, a\rangle, \langle\langle a, b\rangle, b\rangle, \langle\langle b, a\rangle, b\rangle, \langle\langle b, b\rangle, b\rangle\}$
5. $-^J = \{\langle\langle a, a\rangle, a\rangle, \langle\langle a, b\rangle, a\rangle, \langle\langle b, a\rangle, b\rangle, \langle\langle b, b\rangle, a\rangle\}$
6. $=^J = \{\langle a, a\rangle, \langle b, b\rangle\}$
7. $>^J = \{\langle b, a\rangle\}$

and the variable assignment

$$I = \{\langle x, a\rangle, \langle y, b\rangle, \langle z, a\rangle\}.$$

Determine the evaluation of the following formulas:

$f_1 := x \times z = z \rightarrow x = z$

$f_2 := \exists y, z(y \times 0 > 1)$

$f_3 := \neg z - 1 > z \times x \vee \exists x \forall z \exists y(y \times x > x)$

2.2 Clausal normal form (6 pts)

Given the signature $\Sigma_1 = \langle V, F, P, \text{arity} \rangle$ with

$$V = \{x, y, z\}$$

$$F = \{f, g, h\}$$

$$P = \{p, q, r\}$$

$$\text{arity} = \{\langle f, 1 \rangle \, \langle g, 2 \rangle \, \langle h, 1 \rangle \, \langle p, 0 \rangle \, \langle q, 1 \rangle \, \langle r, 2 \rangle\}$$

turn the following formulas into clausal normal form:

$$f_1 := \exists x, z(q(g(x, x)) \vee \exists y(q(x) \wedge p))$$

$$f_2 := \forall y \exists x, z(p \rightarrow p \wedge \neg r(h(z), g(x, y)))$$

$$f_3 := \neg f_2$$

2.3 Unification (6 pts)

Using the above introduced signature Σ_1, determine, if possible, a unifier for the following SSEs:

$$E_1 := \{h(h(z)) \doteq h(y)\}$$

$$E_2 := \{f(f(x)) \doteq z, f(y) \doteq f(h(x)), p \doteq p, x \doteq y\}$$

$$E_3 := \{r(g(f(x), f(z)), z) \doteq r(g(y, f(z)), h(x)), y \doteq f(x), h(x) \doteq z\}$$

2.4 Proof (6 pts)

The president says

> *I am liable for all the individuals not liable for themselves; but I am not liable for those that are liable for themselves.*

Prove that the president is talking nonsense.