## 1 First-order logic (13 pts)

Express the following natural language sentences by means of formulas of firstorder logic:
a) At least one set contains itself.
b) At most one set contains itself.
c) The Russel set contains itself.
d) The Russel set contains all sets not containing themselves.
e) Exactly one of the following is true: $a<b, b<a$, or $a=b$.

In doing so, use exclusively the following predicates/functions:

- $r$ : the constant representing the Russel set
- $c(x, y): \top$ iff the set $x$ contains the set $y$
- $f<h$ : Т iff $f$ is smaller than $h$
- $f=h$ : Т iff $f$ is equal to $h$


## 2 Resolution (9 pts)

Napoleon is developing the strategy for his new campaign. His advisors tell him:
(a) "Prussia will attack if and only if neither Russia nor Prussia will."
(b) "Both Prussia and France will attack."
(c) "That Russia will attack if and only if France will, will happen only in the case that France will attack."

Napoleon is wondering whether
(d) Russia and Poland will attack.

Can you help him find out?
Which of the following statements are true?

1) (d) can be proven based on the advisors' statements.
2) (d)'s negation can be proven based on the advisors' statements.
3) The advisors' statements are inconsistent.
4) The advisors' statements are incomplete.

Justify your answer.

## 3 Intelligent Search (10 pts)

The nodes of the following graph represent certain configurations on a twoplayer game board (gray=you, white=me). It is currently your turn (empty gray node). The values in the nodes represent a reward heuristic (inf stands for $\infty$, i.e., you win; $-\inf$ stands for $-\infty$, i.e., I win).


Depending on how much time you have for your move (you may be playing blitz mode), you can precalculate $n$ of my and your future moves taking the heuristics into account.
i) Which move (a, b, c, or d) is most beneficial for you when you precalculate 1,3 , or 5 moves, respectively?
ii) Do you have a chance to win the game when both of us have enough time to precalculate as many moves as possible? Why?

## 4 Expert and dialog systems (8 pts)

A medical provider wants to build an expert system with the following business logic:

| fever? | high blood pressure? | diarrhea? | diagnosis | number of patients |
| :--- | :--- | :--- | :--- | :--- |
| no | no | no | healthy | 70 |
| no | no | yes | gastroenteritis | 30 |
| no | yes | no | healthy | 60 |
| no | yes | yes | stroke | 90 |
| yes | no | no | flu | 30 |
| yes | no | yes | flu | 30 |
| yes | yes | no | stroke | 0 |
| yes | yes | yes | stroke | 50 |

The system is to be designed such that the average number of asked questions becomes minimal and that the first one is about whether or not the patient has fever.
a) Depending on the response to the first question, which one should be asked next?
b) Draw a decision tree ("call flow") of this expert system.
c) What is the average number of questions asked by the system?

