1 First-order logic (10 pts)

Express the following natural language sentences by means of formulas of first-order logic:

a) *Gauss is smart.*

b) *Mathematicians are smart.*

c) *At least one mathematician is smart.*

d) *Exactly one mathematician is smart.*

e) *If x is negative then $-x$ is positive.*

In doing so, use exclusively the following predicates/functions:

- g: the constant representing Gauss
- $s(x)$: $\top$ iff x is smart
- $m(x)$: $\top$ iff x is a mathematician
- 0: the constant 0
- $f - g$: substracts f from g
- $f = g$: $\top$ iff f equals g
- $f < g$: $\top$ iff f is lower than g

2 Resolution (12 pts)

We are given the following knowledge base:

(a) $A \vee A \wedge C$

(b) $(C \rightarrow B) \rightarrow \neg C$

(c) $A \wedge C \rightarrow A$

Furthermore, we are given the conjecture

(d) $A \rightarrow B \wedge C$

Use the resolution algorithm to prove or disprove (d).

Which of the following statements are true:

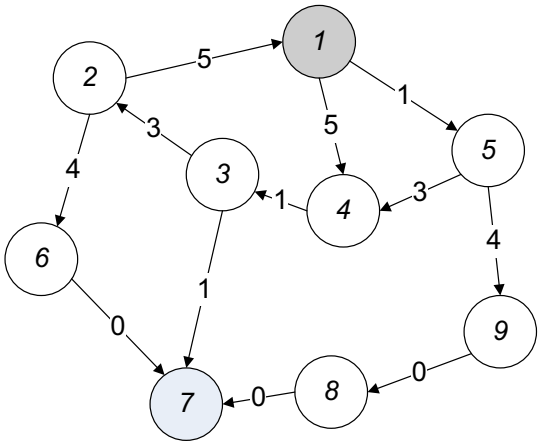
1) (d) can be proven using the knowledge base.

2) (d)'s negation can be proven using the knowledge base.

3) Neither (d) nor (d)'s negation can be proven using the knowledge base.

3 A* (12 pts)

We are given the following graph:



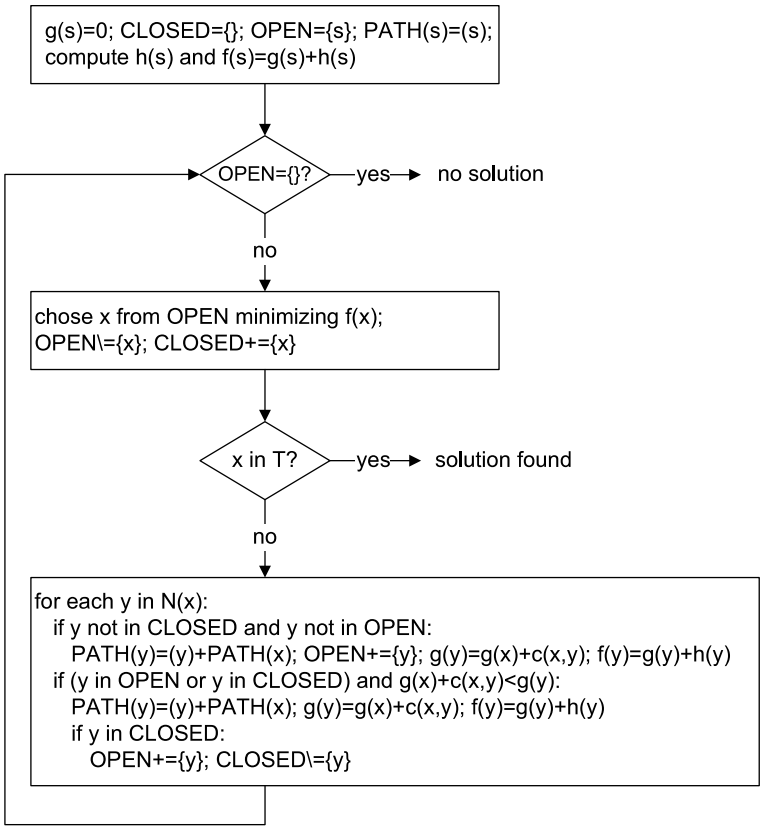
From the figure, we can derive the starting node (1) and the set of target nodes ($T = \{7\}$). Furthermore, we can derive the set of neighbors $N(x)$ for each node x as well as the cost $c(x, y)$ for changing from node x to node y being the arc labels.

Perform an A* search using the following table providing the heuristic estimate $h(x)$:

x	1	2	3	4	5	6	7	8	9
$h(x)$	3	4	0	1	4	2	5	0	1

How do OPEN and CLOSED sets as well as PATH(7) look like when hitting the target?

For this task, use the A* convention introduced in the class as depicted in the following diagram:



4 Expert and dialog systems (6 pts)

The expenses of a large bank's call center are to be minimized. It is known that a call processed by a human agent costs 5 euros on average. Since this seems too expensive, the bank considers the installation of a phone banking system. This system covers only a limited number of standard call reasons (such as account transfers or balance) and therefore achieves a mere 20% automation rate at an average handling time of 300 seconds.

a) How much are the expected savings per call processed by the call center if the dialog system generates costs of 25 cents per minute?

b) Calculate the trade-off parameter T_A.

c) What is the minimum automation rate for which the phone banking system becomes beneficial?